



Reg. No. :

Name :

**Fifth Semester B.Tech. Degree Examination, December 2015
(2008 Scheme)**

08.502 : ADVANCED MATHEMATICS AND QUEUING MODELS (RF)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.



1. Define :
 - I) Basic Solution
 - II) Basic Feasible Solution
 - III) Optimal Solution
2. How do you determine whether (I) an optimal solution is obtained (II) an unbounded solution exists, while solving an LPP by simplex method.
3. Define the terms (I) Total Float (II) Free Float (III) Independent Float.
4. Show that $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 + x_2 = 0 \right\}$ is a subspace of \mathbb{R}^2 .
5. Define null space, column space and row space of an $m \times n$ matrix.
6. Give an example of an orthogonal basis of \mathbb{R}^2 other than the standard basis.
7. Find the change of basis Matrix from the basis $E = \left\{ e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ to the basis $F = \left\{ f_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, f_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$ of \mathbb{R}^2 .



8. State Little's formula for the $M/M/1 : \infty/FIFO$ queuing model.
9. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is distributed exponentially with mean 4 min. Find the average number of customers waiting in the system.
10. Give Formulas of (I) Average number of customers in the queue (II) Average number of customers in the system for the $M/M/C : \infty/FIFO$ model.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) Solve by Simplex method. maximize $Z = 2x_1 + 2x_2 + 4x_3$ subject to $2x_1 + 3x_2 + x_3 \leq 300$; $x_1 + x_2 + 3x_3 \leq 300$; $x_1 + 3x_2 + x_3 \leq 240$ $x_1, x_2, x_3 \geq 0$.
- b) A project has the following time schedule

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Time	2	1	4	8	5	3	1	5	4	3	2

- I) Construct a PERT network
 - II) Find the critical path and project duration.
12. a) Solve the following LPP by Big M method.
minimize $z = 4x_1 + x_2$ subject to $3x_1 + x_2 = 3$; $4x_1 + 3x_2 \geq 6$, $x_1 + 2x_2 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$.
 - b) The three time estimates for the various activities of a small project is given below. Draw the network and find the critical path and the probability of completing the project atleast 15 days earlier than expected.

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Optimistic time	5	1	2	3	1	2	1
Most likely time	6	1	4	6	1	2	4
Pessimistic time	7	7	12	15	1	8	7



Module – II

13. a) Find the Lu decomposition of $A = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}$ and hence solve the system

$$Ax = B, \text{ where } B = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

b) Find a spanning set for the null space of $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$ and hence find the dimension of $\text{nul } A$.

14. a) Show that $v_1 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix}$, $v_2 = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$, $v_3 = \begin{bmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix}$ form an

orthonormal basis of \mathbb{R}^3 . Express the vector $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in \mathbb{R}^3 as a linear

combination of v_1, v_2 and v_3 .

b) Find the singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$



Module – III

15. If people arrive to purchase cinema tickets at an average rate of 6 per minute and it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and if it takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket.
 - a) Can he expected to be seated for the start of the picture ?
 - b) What is the probability that he will be seated for the start of the picture ?
 - c) How easily must he arrive in order to be 99% sure of being seated for the start of the picture.
16. A duplicating machine maintained for office use is operated by an office assistant who earns Rs. 50 per hour. The time to complete each job varies according to an exponential distribution with mean 6 minutes. Assume a poisson input with an average arrival rate of 5 jobs/hour of an 8 hour day is used as a base, determine
 - I) The percentage idle time of the machine
 - II) The average time a job is in the system
 - III) The average earning per day of the assistant.